

Probing Slepton Mass Non-Universality at e^+e^- Linear Colliders

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Abstract

There are many models with non-universal soft SUSY breaking sfermion mass parameters at the grand unification scale. Even in the mSUGRA model scalar mass unification might occur at a scale closer to M_{Planck} , and renormalization effects would cause a mass splitting at M_{GUT} . We identify an experimentally measurable quantity Δ that correlates strongly with $\delta m^2 \equiv m_{\tilde{e}_R}^2(M_{GUT}) - m_{\tilde{e}_L}^2(M_{GUT})$, and which can be measured at electron-positron colliders provided both selectrons and the chargino are kinematically accessible. We show that if these sparticle masses can be measured with a precision of 1% at a 500 GeV linear collider, the resulting precision in the determination of Δ may allow experiments to distinguish between scalar mass unification at the GUT scale from the corresponding unification at $Q \sim M_{Planck}$. Experimental determination of Δ would also provide a distinction between the mSUGRA model and the recently proposed gaugino-mediation model. Moreover, a measurement of Δ (or a related quantity Δ') would allow for a direct determination of δm^2 .

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Theories with multiple scalar fields such as weak scale supersymmetry (SUSY) contain new sources of flavour violation, and so are strongly constrained by experiment. There are, however, a number of mechanisms to suppress these unwanted effects. The most obvious way is to assume that the scalars are heavy. However, heavy scalars potentially destabilize the electroweak symmetry breaking (EWSB) sector, and are generally an anathema within the SUSY context¹. Flavour violation from supersymmetric gauge (and gaugino) interactions is also suppressed if fermion and sfermion mass matrices are aligned, or scalars (at least those with the same gauge quantum numbers) are sufficiently degenerate. Indeed within the extensively studied minimal supergravity (mSUGRA) and gauge-mediated SUSY breaking (GMSB) models, the latter mechanism is used to suppress these flavour violating effects. This is also the case in the more recently proposed anomaly-mediated SUSY breaking (AMSB) and gaugino-mediated SUSY breaking models.²

The mSUGRA Grand Unified Theory (GUT) framework is characterized by the assumption of universality of soft SUSY breaking scalar masses (m_0) and trilinear scalar couplings (A_0), renormalized at some high scale in between M_{GUT} and M_{Planck} . In practice, this scale is frequently taken to be M_{GUT} , though it should be kept in mind that renormalization group evolution from the true scale of unification may lead to significant splitting of scalar masses at $Q = M_{GUT}$ [3]. A similar situation obtains in the simplest gaugino-mediation model [4] based on $SU(5)$ where it is assumed that soft SUSY breaking scalar masses essentially vanish at the so-called compactification scale which is taken to be between M_{GUT} and M_{Planck} . The situation in these scenarios is in sharp contrast to the corresponding situation in GMSB or AMSB models, where there may be a diversity of soft SUSY breaking scalar masses at the high scale.

If the two selectrons and the lighter chargino \widetilde{W}_1 are discovered and their masses along with the mass of the lightest supersymmetric particle (LSP) determined, it would be easy to differentiate mSUGRA, GMSB and AMSB models from one another. Within the GMSB framework, \tilde{e}_L is considerably heavier than \tilde{e}_R while in the AMSB framework, the lighter chargino is essentially degenerate with the $SU(2)$ gaugino-like neutralino LSP. However, differentiating between the mSUGRA model³ with $m_0 \sim \frac{1}{2}m_{1/2}$ and the $SU(5)$ model with gaugino mediation of SUSY breaking is obviously more difficult. Although the conceptual foundations of the two models are very different, they differ quantitatively only due to the additional small splitting between the two selectron masses at the GUT scale. Of course, such a splitting could also arise if the scale of unification of scalar masses differs from M_{GUT} ; for the same value of $m_{1/2}$, such a model cannot be readily differentiated from the gaugino mediation scenario. It is also possible that even within the SUGRA framework $m(\tilde{e}_L)$ and

¹We do not mean to imply that models with heavy scalars are not allowed. Indeed a construction of phenomenologically viable models with heavy scalars and an examination of their viability has received considerable attention in recent years [1].

²For a general overview of models with non-universal SUSY breaking terms, see Ref. [2].

³In the rest of the paper we will define the mSUGRA framework to have universal parameters at $Q = M_{GUT}$.

$m(\tilde{e}_R)$ are split by D -term contributions from additional $U(1)$ factors of the underlying gauge group [5]. The purpose of this paper is to examine how well we can probe deviations from universality of intra-generational slepton mass parameters expected within the mSUGRA framework, if sleptons and charginos are discovered at an electron-positron collider operating at $\sqrt{s} = 500$ GeV and their masses are measured with a precision of $\sim 1\%$ [6–9].

Neglecting electron Yukawa couplings, the one-loop renormalization group equation (RGE) for the difference of selectron masses can be written as,

$$\begin{aligned} \frac{d}{dt} (m_{\tilde{e}_R}^2 - m_{\tilde{e}_L}^2) &= -\frac{2}{16\pi^2} \left(\frac{9}{5} g_1^2 M_1^2 - 3g_2^2 M_2^2 - \frac{9}{10} g_1^2 S \right) \\ &= -\frac{1}{2\pi} \frac{M_2^2}{\alpha_2^2} \left(\frac{9}{5} \alpha_1^3 - 3\alpha_2^3 \right) + \frac{9}{20\pi} \alpha_1 S, \end{aligned} \quad (1)$$

where $t = \ln(Q/M_{GUT})$, and

$$S = m_{H_u}^2 - m_{H_d}^2 + \sum (m_{\tilde{q}_L}^2 - m_{\tilde{\ell}_L}^2 - 2m_{\tilde{u}_R}^2 + m_{\tilde{d}_R}^2 + m_{\tilde{\ell}_R}^2).$$

Here, the sum extends over all the generations. In the last step of Eq. (1), we have assumed that gauge couplings and gaugino masses unify at M_{GUT} . The running gauge coupling $\alpha_i(Q)$ is given at the 1-loop level by,

$$\alpha_i(Q) = \frac{\alpha_i(M_Z)}{1 - \frac{b_i}{2\pi} \alpha_i(M_Z) \ln(\frac{Q}{M_Z})}, \quad (2)$$

with b_i denoting the coefficient of the 1-loop β -function for the i^{th} factor of the gauge group: $b_1 = 33/5$ and $b_2 = 1$. The quantity S that appears above satisfies the 1-loop RGE,

$$\frac{dS}{dt} = \frac{b_1}{2\pi} \alpha_1 S, \quad (3)$$

which can be easily integrated to give,

$$S(Q) = \frac{S_{GUT}}{1 - \frac{b_1}{2\pi} \alpha_1(M_{GUT}) \ln(\frac{Q}{M_{GUT}})} = S_{GUT} \frac{\alpha_1(Q)}{\alpha_1(M_{GUT})}. \quad (4)$$

Notice that at the 1-loop level S_{GUT} , and hence S , vanishes within the mSUGRA framework. It is now straightforward to integrate Eq.(1) between $Q = M_{GUT}$ and $Q = m_{\tilde{e}}$. Noting that the factor M_2^2/α_2^2 on the right hand side is independent of t (at one loop), we only need to evaluate,

$$\int dt \alpha_i^3(t) = \frac{\pi}{b_i} \left(\alpha_i^2(m_{\tilde{e}}) - \alpha_i^2(M_{GUT}) \right),$$

to integrate the first term. We obtain,

$$\begin{aligned} m_{\tilde{e}_R}^2 - m_{\tilde{e}_L}^2 + \frac{M_2^2}{2\alpha_2^2(M_2)} \left[\frac{3}{11} \left(\alpha_1^2(m_{\tilde{e}}) - \alpha_1^2(M_{GUT}) \right) - 3 \left(\alpha_2^2(m_{\tilde{e}}) - \alpha_2^2(M_{GUT}) \right) \right] = \\ \delta m^2 + \left(\frac{1}{2} - 2 \sin^2 \theta_W \right) M_Z^2 \cos 2\beta - \frac{9}{10b_1} S_{GUT} \left(1 - \frac{\alpha_1(m_{\tilde{e}})}{\alpha_1(M_{GUT})} \right), \end{aligned} \quad (5)$$

where $\delta m^2 \equiv m_{\tilde{e}_R}^2(M_{GUT}) - m_{\tilde{e}_L}^2(M_{GUT})$ as well as S_{GUT} vanish within the mSUGRA framework. The slepton masses on the left hand side of Eq. (5) are the running masses evaluated at the slepton mass scale, and are to a very good approximation the physical masses of the selectrons. The middle term on the right hand side of (5) comes from the usual hypercharge D -terms.

We note that except for M_2 , all the quantities on the left-hand side of Eq. (5) can be directly measured, or like the running gauge couplings, directly obtained from measured experimental quantities⁴ using Eq. (2). Furthermore, the right hand side of Eq. (5) is very small ($\sim 0.04 M_Z^2 \cos 2\beta \lesssim (20 \text{ GeV})^2$) if scalar masses are universal, but not necessarily so otherwise.

Within the mSUGRA framework, the lighter chargino is essentially always dominantly an SU(2) gaugino. This suggested to us that we should examine the “directly measurable quantity”,

$$\Delta = m_{\tilde{e}_R}^2 - m_{\tilde{e}_L}^2 + \frac{m_{\tilde{W}_1}^2}{2\alpha_2^2(m_{\tilde{W}_1})} \left[\frac{3}{11} \left(\alpha_1^2(m_{\tilde{e}}) - \alpha_1^2(M_{GUT}) \right) - 3 \left(\alpha_2^2(m_{\tilde{e}}) - \alpha_2^2(M_{GUT}) \right) \right], \quad (6)$$

which should also be small within mSUGRA as long as, (i) the chargino mass is not very different from M_2 , and (ii) higher loop contributions to the RGEs are not large. It should, of course, be kept in mind that even if the difference in (i), or the change in sparticle masses in (ii), is at the level of just 1-2%, the near perfect cancellations implied by Eq. (5) will no longer obtain, resulting in a large *relative* change in Δ from its 1-loop value of $\sim 0.04 M_Z^2 \cos 2\beta \lesssim (20 \text{ GeV})^2$. Nevertheless, even with these corrections, we expect $|\Delta| \ll m_{\tilde{e}_L}^2, m_{\tilde{e}_R}^2, m_{\tilde{W}_1}^2$. Just how well Δ can be used to measure the (non-)universality of slepton masses forms the remainder of this paper.

We begin by first examining the range of Δ within the mSUGRA framework. Toward this end, we randomly generate mSUGRA models within the parameter range,

$$\begin{aligned} 0 &< m_0 < 600 \text{ GeV}, \\ 100 \text{ GeV} &< m_{1/2} < 600 \text{ GeV}, \\ 2 &< \tan \beta < 50, \\ -2m_0 &< A_0 < 2m_0, \\ \mu &= +, -, \end{aligned} \quad (7)$$

and compute the sparticle masses using the two loop RGEs incorporated into the program ISAJET [10]. We first check whether these satisfy the lower limits on sparticle masses obtained from experiments at LEP. We take these to be 100 GeV for selectrons or charginos, 85 GeV for $\tilde{\tau}_1$ and 90 GeV for h the lighter CP even scalar [11]. Moreover, if $\tan \beta \leq 8$, we require $m_h \geq 112 \text{ GeV}$ [11]. We only keep models that (i) satisfy these experimental constraints, (ii) break electroweak symmetry radiatively, and (iii) have a neutralino LSP in our analysis. For each model that we retain, we compute Δ from the chargino and electron masses and the *1-loop couplings* as given by Eq. (2), and show this versus the chargino

⁴The scale M_{GUT} is defined to be the scale at which the couplings α_1 and α_2 meet.

mass in the scatter plot in Fig. 1a. We show a model with $\mu > 0$ ($\mu < 0$) by a dark plus (light cross). Where the light crosses and dark pluses overlap, just the latter are visible. The “parabolic curve”, however, shows the lower limit of Δ for models with negative μ . In Fig. 1b, we once again show Δ versus the chargino mass, but this time restrict ourselves to models with $\mu > 0$ where both selectrons and the lighter chargino are lighter than 250 GeV, *i.e.* kinematically accessible at a 500 GeV e^+e^- collider that is being considered for construction some time in the future. Fig. 1c shows the same scatter plot, but for negative values of μ .

We see that the magnitude of Δ is small compared to the scale of particle masses as we had anticipated. Furthermore, restricting our attention to frames *b* and *c*, *i.e.* to the situation where Δ would be measurable at a 500 GeV e^+e^- collider, we see that the range of Δ is considerably diminished. Indeed models with positive Δ appear to be possible only for heavy sleptons, which may only be accessible at a higher energy collider. Much more interesting to us is the fact that in frames *b* and *c*, the overlap (for a given chargino mass) in the allowed range of Δ between models with positive and negative μ is much reduced. We will use this feature later in our analysis.

To get a better feel for the quantity Δ , we have examined the various factors that cause it to differ from zero, its expected 1-loop value (aside from the small D -term) within the mSUGRA framework. For frames *b* and *c*, by far the largest factor is the replacement of M_2 in Eq. (5) by the chargino mass. The use of two loop RGEs to evaluate sparticle masses yields a spread in Δ that is, depending on the chargino mass, 5-10 times smaller than the spread in Fig. 1b or *c*. Moreover, since the limit on m_h essentially excludes $\tan\beta \leq 4$, $\cos 2\beta \lesssim -0.9$, and hence, the D -term contribution lowers the value of Δ by an essentially constant value of about 300 GeV², regardless of other parameters. This then implies that if M_2 can be reliably determined from the data, the expected spread in Δ will be much reduced⁵ from that shown in the figure.

We now turn to an examination of how well Δ can be used to discriminate mSUGRA from models with non-universal slepton masses at M_{GUT} . To introduce such a non-universality, we modify the mSUGRA boundary conditions by choosing the GUT scale mass parameter for all three flavours of right handed sleptons to be $m_{\ell_R}^2 = m_0^2 + \delta m^2$, but leave all other scalar masses at m_0 . This then ensures that no dangerous lepton flavour violation is induced [12]. For the simple parametrization that we have introduced above, S_{GUT} in Eq. (5) is just $3\delta m^2$. For $\delta m^2 = 0$, our model reduces to the mSUGRA model with unification of scalar masses at $Q = M_{GUT}$.

The new parameter δm^2 that we have introduced is positive for an mSUGRA $SU(5)$ model where scalar masses unify at a scale $Q > M_{GUT}$. It is also positive for the gaugino-mediated SUSY breaking model. However, since we do not want to commit to any specific form for the underlying physics at the high scale, we analyze both signs of δm^2 .

To study the effect of slepton mass non-universality on Δ , we have randomly generated such models, taking $-(200 \text{ GeV})^2 \leq \delta m^2 \leq (200 \text{ GeV})^2$, with other parameters in the range

⁵This is only true for models with sleptons lighter than 250 GeV. For the case of all models shown in Fig. 1a, the spreads due to two loop contributions (with scalars heavy) is comparable to that from the approximation $M_2 \simeq m_{\tilde{W}_1}$.

given by (7). We require that at $Q = M_{GUT}$, (i) $m_{\tilde{\ell}_R}^2 > 0$ and (ii) $m_{\tilde{\ell}_R} - m_{\tilde{\ell}_L} \leq 100$ GeV. As before, we only accept models that satisfy the experimental and theoretical constraints discussed previously, and for which the two selectrons and the lighter chargino are all lighter than 250 GeV. We then compute Δ for each of the models that we accept and plot the result versus the non-universality parameter δm^2 in Fig. 2a. As in Fig. 1, we show models with positive (negative) values of μ by dark pluses (light crosses). The crosses are again not visible when they lie under the pluses: the dashed line marks the boundary below which there are no crosses. To show the spread of the models more clearly, we have split into two the long diagonal band along which all the models lie. The band on the left shows the models with predominantly negative values of δm^2 (upper scale) which result in mostly negative values of Δ (left hand scale). The band on the right includes the remaining models, *i.e.* mainly models with positive values of δm^2 (lower scale) which mostly yield $\Delta > 0$ (right hand scale). The horizontal bars show the limits of the range of Δ within the mSUGRA model, $-5250 \text{ GeV}^2 \lesssim \Delta(\text{mSUGRA}) \lesssim -750 \text{ GeV}^2$.

Several features of this figure are worthy of note.

1. There is a very strong correlation between Δ and δm^2 . This is to be expected, since the first and last terms on the right hand side of Eq. (5) are each proportional to δm^2 while the middle term is small and, as discussed above, approximately constant.
2. Eq. (5) suggests that the slope of the band would be unity but for the last term that causes it to be smaller. It is easy to check that for slepton masses ~ 100 GeV, the reduction in the slope is about 8% for each generation of sleptons with an intra-generational splitting of δm^2 . The slope of the band in the figure agrees remarkably well with our expectation for three split slepton families.
3. The width of the bands, *i.e.* the spread in the values of Δ , is essentially independent of δm^2 . This is understandable once we recognize that even in these models, the replacement of M_2 by $m_{\tilde{W}_1}$ is the largest source of the spread of Δ . This is because the additional terms in Eq. (5) that were absent in mSUGRA are insensitive to model parameters. Clearly the first term on the right hand side is completely independent, and the last term is only logarithmically dependent on the slepton mass.⁶

It is clear that if Δ lies significantly outside its mSUGRA range, it should be possible to use it to distinguish models with non-universal slepton masses from mSUGRA. The question then is, ‘‘How well can Δ be determined?’’. Noting that the coefficient of the $m_{\tilde{W}_1}^2$ term in the definition (6) of Δ is very close to 1/2, we can write the error in Δ as,

$$\begin{aligned} \delta\Delta &= 2 \left[m_{\tilde{e}_R}^2 (\delta m_{\tilde{e}_R})^2 + m_{\tilde{e}_L}^2 (\delta m_{\tilde{e}_L})^2 + \frac{1}{4} m_{\tilde{W}_1}^2 (\delta m_{\tilde{W}_1})^2 \right]^{\frac{1}{2}} \\ &\simeq 3 \left(\frac{\delta m}{m} \right) m^2, \end{aligned}$$

⁶To complete the argument we should also note that, since slepton masses do not enter the chargino mass matrix, the modification of allowing unequal slepton masses at the GUT scale does not significantly alter $M_2 - m_{\tilde{W}_1}$.

where in the last step we have assumed that the selectrons and charginos have the same mass (m), and the same relative error in the mass measurement. The *maximum* error in Δ which occurs when the sparticles are all 250 GeV, is 1875 GeV^2 , assuming that selectron and chargino masses are measured with a precision of 1%. For sparticle masses $\sim 150 \text{ GeV}$, $\delta\Delta \simeq 675 \text{ GeV}^2$. Allowing for a typical error $\sim 1000 \text{ GeV}^2$ on Δ , we see from Fig. 2a that without any other information it should be possible⁷ to detect intra-generational non-universality of GUT scale slepton masses if $|\delta m^2| \gtrsim 7000 \text{ GeV}^2$.

It is, however, obvious that we can do better if we can reduce the spread of Δ within the mSUGRA framework; this will also correspondingly reduce the width of the band. A look at Fig. 1b and Fig. 1c immediately shows that the spread in Δ is considerably reduced once the chargino mass is known. To illustrate this, keeping in mind that the chargino mass can be determined [6–9] at the percent level, we plot Δ versus δm^2 in Fig. 2b, but only for those models with $150 \leq m_{\tilde{W}_1} \leq 160 \text{ GeV}$. The slanted solid lines show the boundaries of the full band in frame *a*. We see that using the chargino mass information⁸ together with Δ should allow for detection of non-universality if $|\delta m^2| \gtrsim 5000 \text{ GeV}^2$. For $m_0 \sim 120 \text{ GeV}$, this corresponds to a splitting of about 15%. We have checked that unlike the chargino mass, restricting the selectron mass to be close to its measured value does not reduce the spread of the band, and so does not help to increase the range over which non-universality might be detectable. Presumably, this is because most of this spread comes from the difference $M_2 - m_{\tilde{W}_1}$ which is insensitive to the selectron mass.

Can we improve upon this? Improving the precision with which sparticle masses are measured will reduce the measurement error on Δ but not the width of the band in Fig. 2b, and so does not help a great deal as the latter is the major source of the uncertainty. We see from the figure that the width is indeed reduced, along with the mSUGRA range from Fig. 1, if the sign of μ can be determined. There are mSUGRA studies [6,8] that suggest that this should be possible at linear colliders via careful measurements of the chargino production channel. Although these studies have only been done within the mSUGRA framework, the conclusions regarding the properties of charginos and neutralinos that have been obtained should also hold for the present case since the introduction of δm^2 does not affect chargino and neutralino mass matrices except via loop corrections. If the sign of μ is indeed determined, the range of δm^2 over which the model with non-universality may be confused with mSUGRA is reduced: For instance, for $\mu < 0$ and $m_{\tilde{W}_1} \sim 150 \text{ GeV}$, from Fig. 1c and Fig. 2b, we can infer that the two models should be distinguishable except when $|\delta m^2| \lesssim 4000 \text{ GeV}^2$.

Yet another possibility for improving the sensitivity of Δ arises if the gaugino mass parameter M_2 can directly be determined from the measured chargino and neutralino masses. We would then not need the replacement of M_2 by the chargino mass and the width of our band would be greatly reduced. This is illustrated in Fig. 3 where we plot Δ' , *defined* to be

⁷To obtain this we linearly combine the 1σ error in the measurement of Δ to the uncertainty from the width of the band.

⁸Reducing the mass window further does not reduce the width of the band significantly, as can be seen from Figs. 1b and c.

the left hand side of Eq. (5), against δm^2 for the same models as in Fig. 2a. In this case, the positive and negative μ cases essentially overlap. The slanted lines are the boundaries of the corresponding band in Fig. 2a. We see that the Δ' band is shifted upwards relative to the Δ band. This is because $M_2 \geq m_{\tilde{W}_1}$ for all these models.⁹ We have checked that within the mSUGRA framework, Δ' is negative and larger than -850 GeV^2 , as shown by the horizontal lines. While the greatly reduced mSUGRA range of Δ' makes it appear that we should be able to probe much smaller values of δm^2 , we should be cautious since measurement errors which were previously smaller than the uncertainties from the width of the band may now be the dominant limitation.

To assess the usefulness of Δ' , we rely on previous studies that examine how well the gaugino mass parameters may be determined. We recognize that the extraction of the underlying gaugino masses could depend on where we are in parameter space. Since only few case studies have been performed, we should view the precision that we quote below only as representative of what might be attainable. In their assessment of how well gaugino mass unification could be tested at a linear collider, Tsukamoto *et al.* [6] have shown that even for charginos as heavy as 220 GeV, it would be possible to measure M_2 with a precision of about 10%, while the parameter M_1 could be determined with a precision of 3%. This was a result of a global fit to the chargino and lightest neutralino masses, and slepton and chargino production cross sections with polarized beams. Although they treated M_1 and M_2 as independent parameters, assuming the gaugino mass relation between these would imply that M_2 would also be determined to at least 3%. An mSUGRA case study done at the 1996 Snowmass Workshop [8] illustrates an example where M_2 is determined with a precision of 1.5%. Although this particular case which had a chargino of just 97 GeV is now excluded by the LEP data, we present it here to illustrate the precision that might be attainable at these facilities. These studies all assume that just the lighter chargino is kinematically accessible. If the heavier chargino is also accessible, the analysis of Ref. [13] suggests that M_2 and also μ might be extracted with a precision of $\sim 1\%$, assuming only statistical errors corresponding to an integrated luminosity of 1000 fb^{-1} . Precision measurements may also be possible at the Large Hadron Collider, at least within the mSUGRA framework. Indeed several cases are shown in Ref. [14] where the unified gaugino mass parameter is claimed to be determined with a precision of 1-2%. For all the cases analyzed there, $m_{1/2}$ is claimed to be determined to better than 10%. In models with unified gaugino masses, it is not unreasonable to suppose that M_2 would be determined with a comparable precision. We stress though that these analyses determine $m_{1/2}$ and not the gaugino masses independently, and also it is only for favourable ranges of parameters that the very precise determination of the gaugino masses is possible.

If slepton masses are determined to 1%, and M_2 assumed to be determined to (1,2,5,10)%, then the corresponding error on Δ' is about (700, 800, 1300, 2400) GeV^2 for M_2 and selectron masses of 150 GeV. We see from Fig. 3 that a 1% measurement of M_2 allows for a distinction between mSUGRA and the non-universal model for $|\delta m^2|$ as small as 1900 GeV^2 , while measurement of M_2 with a precision of 5% (10%) degrades this to about 2700 GeV^2 (4100 GeV^2). It thus appears that an M_2 determination with a precision of about 10% is

⁹This is not true in general.

about as effective as using Δ , assuming that the sign of μ can be determined. It should, of course be remembered that the uncertainty in Δ' scales quadratically as the sparticle masses, so that the range of δm^2 over which it will be possible to probe non-universality would be correspondingly smaller (larger) if the sparticles are lighter (heavier) than 150 GeV.

We have attempted to leave our analysis as model independent as possible. Nonetheless, it is interesting to ask what values Δ (or Δ') might take in specific models, and whether these might be probed by this type of analysis. We note that Δ in GMSB and AMSB models differs qualitatively from its value within the mSUGRA framework. But since these models have features that make them easily distinguishable from mSUGRA, we focus on the “much more mSUGRA-like” models such as mSUGRA with scalar mass unification at M_{Planck} or gaugino-mediation. It is not our purpose to analyze these in detail here, but we observe that in representative examples in the literature (Fig. 3 of Ref. [3] or Fig. 1 of Ref. [2]) a GUT scale splitting $\sim 10 - 15\%$ appears possible for SUGRA models with scalar mass unification at M_{Planck} . For the gaugino-mediation model, Fig. 6 of Ref. [2] suggests that $\tilde{\ell}_L$ and $\tilde{\ell}_R$ masses may be split by almost 20% at $Q = M_{GUT}$. Our analysis shows that splittings of this magnitude should, quite possibly, be discernible in experiments at the linear collider.

Another model with split $\tilde{\ell}_L - \tilde{\ell}_R$ mass parameters that has been recently examined is the $SO(10)$ SUSY GUT with D -terms. In this case, δm^2 is an independent parameter equal to $4M_D^2$ in the notation of Ref. [15]. In this study, it has been shown that reasonable phenomenology can be obtained for $M_D \sim \frac{1}{3}m_0$. A slepton mass splitting of this magnitude should certainly be detectable.

Before moving further, we mention that a recent study [16] has claimed that by studying the threshold behaviour of the selectron and chargino production cross sections, it should be possible to measure these masses with a precision of $\sim 0.1\%$, assuming an integrated luminosity of 100 fb^{-1} for each measurement. The same study also suggests that M_2 would be determined with a comparable precision. Without making any representation about whether such precision might actually be attainable, we merely note that if this precision is attained, it will be possible to probe slepton mass non-universality for $|\delta m^2|$ as small as 1000 GeV^2 via a determination of Δ' .

Up to now, our focus has been on whether models with non-universal slepton masses can be distinguished from the mSUGRA model. It should be clear that experimental determination of Δ or Δ' can also directly be used to extract the slepton mass splitting parameter δm^2 . Just how well this can be done depends on what sparticle masses turn out to be, since these reflect directly on the precision with which Δ or Δ' can be determined. As an illustration, we consider that the selectrons and the chargino each have a mass $\sim 150 \text{ GeV}$, and that this is determined to within 1%. In this case Δ will be measured with an error of about $\pm 700 \text{ GeV}^2$. Fig. 2b then shows that the corresponding precision with which δm^2 may be determined is about $\pm 2500 \text{ GeV}^2$ (and somewhat better if the sign of μ is also determined).

The precision with which δm^2 might be determined via a measurement of Δ' is somewhat better. For the same mass values as before, δm^2 may be determined to within $\pm 1400 \text{ GeV}^2$ ($\pm 2200 \text{ GeV}^2$) if M_2 can be determined to within 1% (5%). Of course, since the Δ' band is much narrower, the actual precision will be more sensitive to the actual values of sparticle masses which govern the error on Δ' .

We should compare our approach to (non-)universality to that in Ref. [17]. In this study the values of sparticle masses obtained from measurement were evolved to the GUT

scale (using two loop RGEs) to see if they converged to a common value within errors. This approach requires that all sparticle masses be determined. For this, an integrated luminosity of 1000 fb^{-1} and a centre of mass energy up to 1 TeV was required (to be able to measure squark masses which are taken to be smaller than 500 GeV in this analysis). The main purpose of this study was to examine how well the universality of scalar and gaugino masses could be tested. It was assumed that first generation slepton, squark and chargino masses would be measured by scanning the threshold behaviour of the cross section with a precision of $\sim 0.1\%$ [16]. It was shown that these measurements would convincingly illustrate the underlying unification of scalar, as well as of gaugino, masses at the GUT scale within the mSUGRA framework. It was not the purpose there to examine how well these experiments could probe non-universality and so, of course, there was no attempt to examine if δm^2 could be determined.

Our study focuses on non-universality among the sleptons, and makes no attempt to check the unification of slepton masses with squark or Higgs boson masses. In contrast to Ref. [17], measurements of just the two selectron and chargino masses are used in our analysis. Finally, we also quantify how well the slepton mass splitting at the GUT scale will be determined by these measurements.

Before concluding, we point out one more feature about the quantity Δ that is unrelated to the main subject of this paper. When examining the correlation between Δ and other mSUGRA parameters, we found that there is a surprising correlation with A_0 . This is illustrated in Fig. 4 where we show Δ versus A_0/m_0 for the set of mSUGRA models in Fig. 1b,c, but with the chargino mass in the range $150\text{ GeV} \leq m_{\tilde{W}_1} \leq 160\text{ GeV}$, and (a) $m_{\tilde{e}_R} \leq 140\text{ GeV}$, and (b) $180\text{ GeV} \leq m_{\tilde{e}_R} \leq 220\text{ GeV}$. As before, models with positive (negative) values of μ are denoted by a dark plus (light cross). Assuming that sparticle masses are measured with a precision of 1%, the measurement error in Δ is expected to be $\sim 500\text{ GeV}^2$ and $\sim 1200\text{ GeV}^2$, respectively. This is insufficient to pin down A_0/m_0 with any precision. If, however, the results of the threshold studies hold up, and mass measurements with an order of magnitude better precision indeed turn out to be possible, then Fig. 4b suggests that it may be possible to determine A_0/m_0 with a precision of about $\pm \frac{3}{4}$ to ± 1 , if the selectron is heavy enough and the sign of μ is known. While this determination is crude, and only possible if sparticle masses happen to lie in a favourable range, we thought it worthwhile to point it out since we are not aware of any other way to get at this parameter.¹⁰

We have checked that Δ' is relatively insensitive to A_0/m_0 . The correlation in Fig. 4 arises because $M_2 - m_{\tilde{W}_1}$ is correlated to A_0/m_0 . We have traced this to the fact that the parameter $m_{H_u}^2$ in the renormalization group improved 1-loop effective Higgs potential is strongly correlated with A_0/m_0 . The electroweak symmetry breaking condition then leads to a correlation between μ (and since it appears in the chargino mass matrix, also $m_{\tilde{W}_1}$) and A_0/m_0 . Indeed if M_2 and $m_{\tilde{W}_1}$ can both be measured at the 0.1% level, their mass difference can be used to pin down A_0/m_0 with a precision similar to that attained via a determination of Δ assuming $m_{\tilde{W}_1}$ and the two selectron masses are measured to within 0.1%. Of course,

¹⁰Measurements of third generation sparticle masses may yield information about the corresponding parameter at the weak scale, but the connection with A_0 is not clear.

if both \widetilde{W}_1 and \widetilde{W}_2 are accessible, then the extracted value of μ [13] may provide a better handle on A_0/m_0 .

Summary: We have studied whether experiments at a future electron-positron collider operating at $\sqrt{s} = 500$ GeV might be able to probe an underlying non-universality in intra-generation slepton masses at the GUT scale. Toward this end, we have identified a new quantity Δ whose value can be directly determined from experiments, and which, as can be seen in Fig. 2 correlates very strongly with δm^2 , the difference of \tilde{e}_L and \tilde{e}_R mass squared parameters at the GUT scale. The value of Δ would be very sensitive to the underlying mechanism of SUSY breaking, and would differ dramatically between the mSUGRA, GMSB and possibly also AMSB frameworks. Of course, these frameworks should be easy to distinguish from one another once supersymmetric particles are discovered. The point, however, is that Δ (or Δ') would be a sensitive probe of slepton non-universality. We have shown that with a 1% measurement of chargino and slepton masses, which is feasible with an integrated luminosity of 20-50 fb^{-1} , it should be possible to detect slepton mass non-universality via a measurement of Δ if $|\delta m^2| \gtrsim 5000$ GeV². If SUSY parameters are in a range that allow for a determination of M_2 (or equivalently M_1) with a precision of 1-2% (5%), then a determination of Δ' would allow non-universality to be probed even if $|\delta m^2|$ is as small as 1900 GeV² (2700 GeV²) for sparticle masses ~ 150 GeV. This should make it possible to probe slepton mass non-universality as predicted by various models in the literature. A measurement of the parameters Δ or Δ' also allows a determination of δm^2 to a precision of $\sim \pm 2500$ GeV², depending on model parameters.

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FIGURES

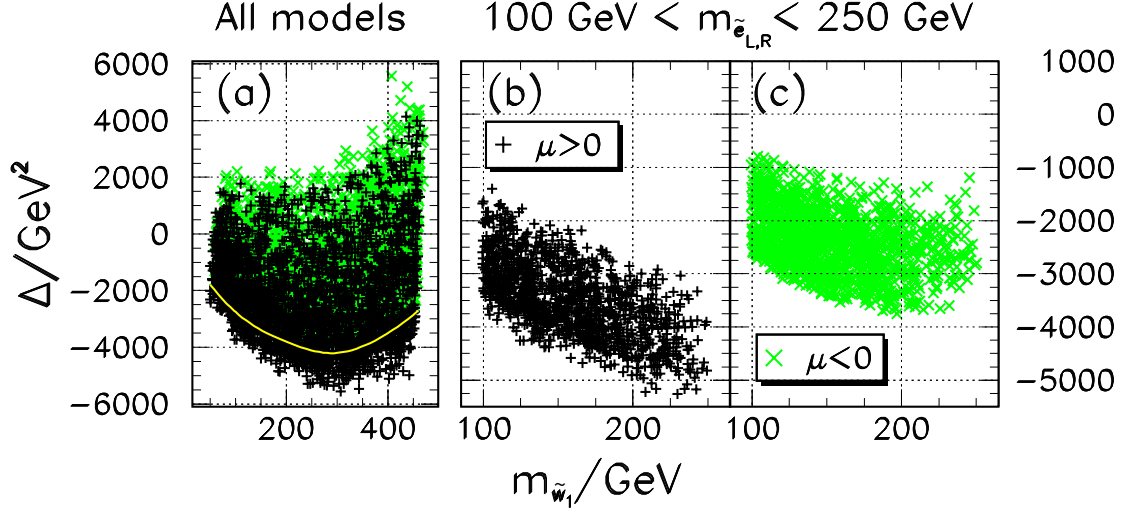


FIG. 1. A scatter plot of Δ , defined in Eq. (6), versus the chargino mass. In frame (a) we show Δ for all the generated mSUGRA models that satisfy the experimental and theoretical constraints discussed in the text. The light crosses and dark pluses denote models with negative and positive values of μ , respectively. Where these overlap, just the dark pluses are visible. The white curve shows the boundary of the band with $\mu < 0$. Frame (b) shows the same thing except that \tilde{e}_L , \tilde{e}_R and \tilde{W}_1 are each also required to be lighter than 250 GeV, with $\mu > 0$. Frame (c) is the same as frame (b) except that $\mu < 0$. Notice that the vertical scale is on the right for frames (b) and (c).

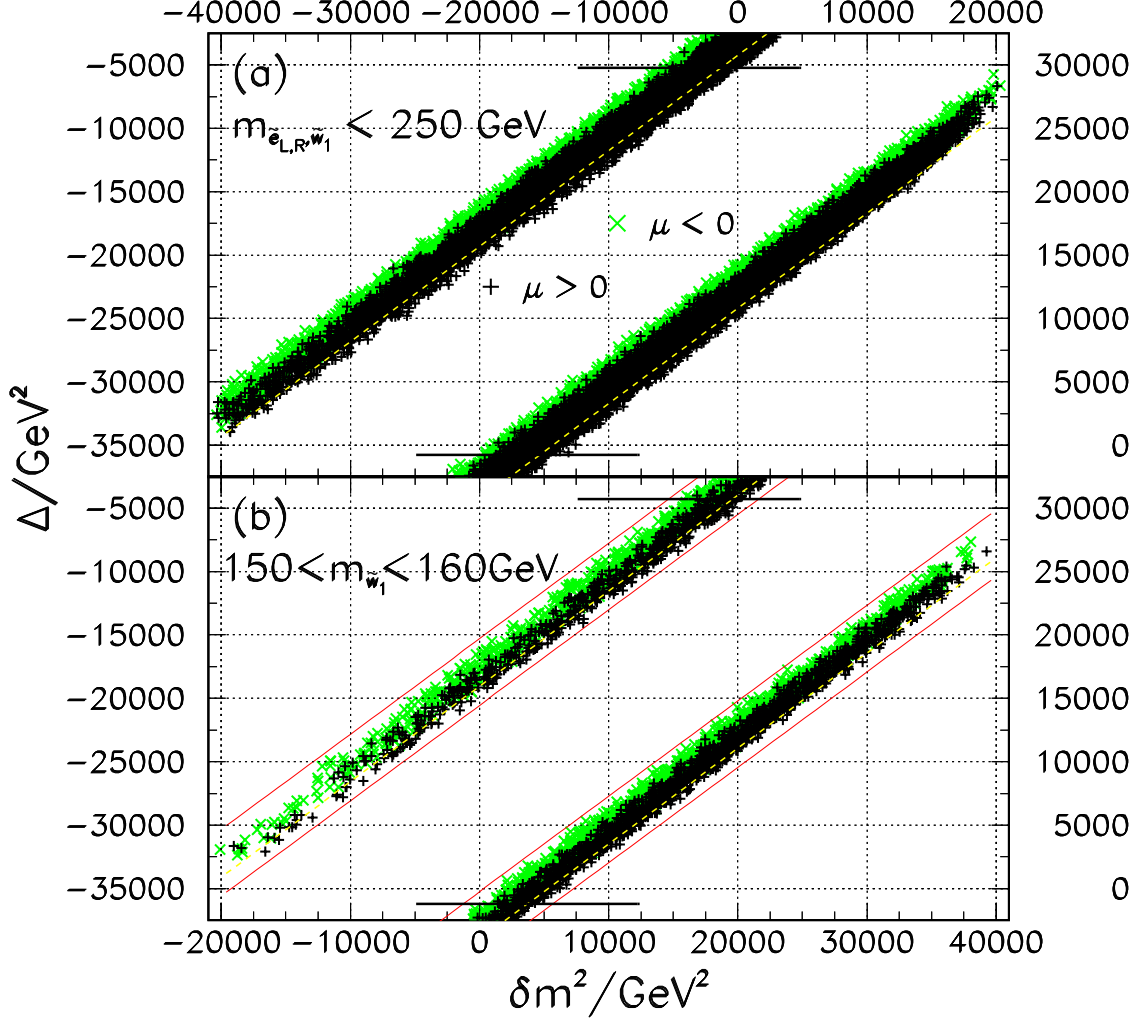


FIG. 2. A scatter plot of Δ versus $\delta m^2 \equiv m_{\tilde{e}_R}^2(M_{GUT}) - m_{\tilde{e}_L}^2(M_{GUT})$. In frame (a) we show Δ for the non-universal models satisfying the same experimental and theoretical constraints as in Fig. 1b,c. Models with positive (negative) values of μ are shown by a dark plus (light cross). Where these overlap, just the dark pluses are visible. The dashed line shows the lower boundary of the region with light crosses. To expand the width of the band in which all the models lie, we have broken the scale into two. The upper band shows Δ for negative values of δm^2 shown on the upper scale, while the lower band shows Δ (vertical scale on the right) for positive values of δm^2 (lower scale). The horizontal lines show the limits on Δ within the mSUGRA framework. Frame (b) shows the same scatter plot as in frame (a), except that the chargino mass is also required to lie between 150 GeV and 160 GeV. The solid lines show the boundaries of the band in frame (a) above while the dashed line shows the boundary of the new region with $\mu < 0$.

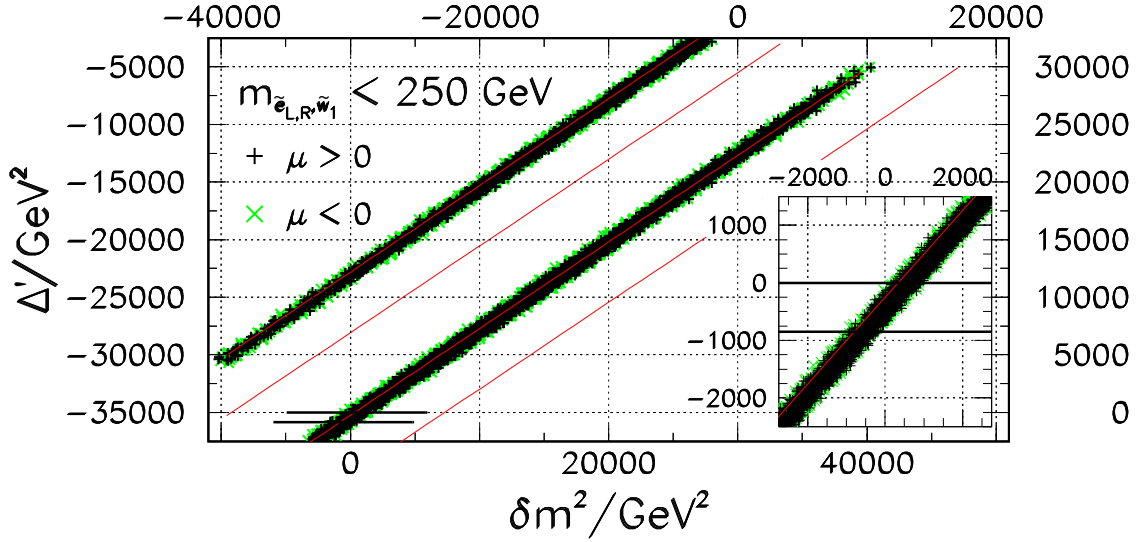


FIG. 3. A scatter plot of the quantity Δ' versus δm^2 defined by the left hand side of Eq. (5) for the same models as in frame (a) of Fig. 2. The bands with positive and negative values of μ essentially overlap so that the light crosses are not visible. The slanted lines show the boundary of the bands in Fig. 2a. The horizontal lines show the bounds on Δ' within the mSUGRA framework. The inset shows a blow-up of the neighborhood of the mSUGRA region.

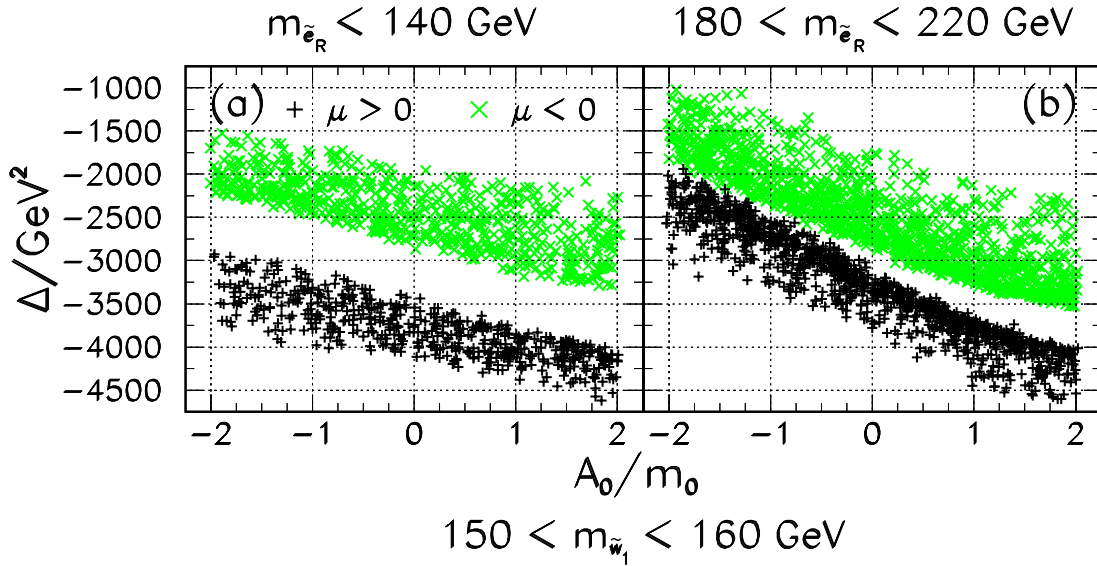


FIG. 4. A scatter plot of the quantity Δ versus A_0/m_0 for mSUGRA models satisfying all constraints in Fig. 1b,c and for which $150 \text{ GeV} \leq m_{\tilde{W}_1} \leq 160 \text{ GeV}$ and (a) $100 \text{ GeV} \leq m_{\tilde{e}_R} \leq 140 \text{ GeV}$, and (b) $180 \text{ GeV} \leq m_{\tilde{e}_R} \leq 220 \text{ GeV}$. Models with a positive value of μ are denoted by a dark plus while those with a negative value of μ are denoted by a light cross.